CHAPTER 10 NOTES – QUADRATIC EQUATIONS

Exercise 10A – Equations of the form $x^2 = k$

Consider $x^2 = 4$.

If we solve for x, we know one solution will be $x = 2$ (because $2 \times 2 = 4$).

However, there is a second solution: $x = -2$ (because $-2 \times -2 = 4$).

So, in most cases, there are **two real solutions** to the equation $x^2 = k$.

Some exceptions to this:

- If $x^2 = 0$, then there is only **one solution**: $x = 0$.
- If $x^2 = a\ negative\ number$, then there are **no real solutions**.
  (no real number when multiplied by itself will equal a negative number)

So, the rule is:

If $x^2 = k$, then

$x = \pm\sqrt{k}$	if $k > 0$
$x = 0$	if $k = 0$
there are $no\ real\ solutions$	if $k < 0$

Example: Solve for x:

a. $x^2 = 9$

$x = \pm\sqrt{9}$

$x = \pm 3$

b. $x^2 = 7$

$x = \pm\sqrt{7}$

c. $x^2 + 3 = 6$

$x^2 = 6 - 3$

$x^2 = 3$

$x = \pm\sqrt{3}$

d. $3 - 2x^2 = 7$

$-2x^2 = 7 - 3$

$-2x^2 = 4$

$x^2 = \frac{4}{-2}$

$x^2 = -2$

$\therefore$ no real solutions

e. $5x^2 + 1 = 51$

$5x^2 = 51 - 1$

$5x^2 = 50$

$x^2 = \frac{50}{5}$

$x^2 = 10$

$x = \pm\sqrt{10}$

Example: Solve for x:

a. $(x-2)^2 = 25$

$(x-2) = \pm\sqrt{25}$

$x - 2 = \pm 5$

$x = 5+2$ or $x = -5+2$

$x = 7$ or $x = -3$

b. $(x+1)^2 = 7$

$(x+1) = \pm\sqrt{7}$

$x = -1 \pm \sqrt{7}$

Exercise 10A: *page 193*
Questions: *all*

Exercise 10B – The Null Factor Law

The Null Factor Law states that:

When the product of two or more numbers is zero*, then AT LEAST one of the numbers must be* zero*.*

So, if $ab = 0$, then either $a = 0$ or $b = 0$ (or both, but we don't need to say that each time).

Example: Solve for x:

a. $x(x+1) = 0$

$x = 0$ or $x+1 = 0$

$x = 0$ or $x = -1$

b. $2x(x-4) = 0$

$2x = 0$ or $x - 4 = 0$

$x = \frac{0}{2}$ or $x = 4$

$x = 0$ or $x = 4$

c. $(x+3)(2x-5) = 0$

$x + 3 = 0$ or $2x - 5 = 0$

$x = -3$ or $2x = 5$

$x = -3$ or $x = \frac{5}{2}$

d. $-6x(2-7x) = 0$

$-6x = 0$ or $2 - 7x = 0$

$x = 0$ or $-7x = -2$

$x = 0$ or $x = \frac{-2}{-7}$

$x = 0$ or $x = \frac{2}{7}$

Exercise 10B: *page 194*
Questions: *all*

Exercise 10C – Solutions by Factorisation

Factorisation! What's that you say? We've done this before? Why yes, yes, we have. This is great for you, because it means you get to revise and learn something new; multitasking at its finest.

The steps to help us solve quadratic equations:

1. RHS = 0

   We can ONLY use the null factor law if the equation $= 0$

   REMEMBER – the null factor law states that if $ab = 0$*, then* $a = 0$ *or* $b = 0$

2. Fully factorise – remember the different types of factorising

   i. Check for a HCF

   ii. DOTS

   $x^2 - a^2 = (x+a)(x-a)$

   iii. Perfect square

   $x^2 + 2ax + a^2 = (x+a)^2$

   iv. Sum and product method

   Normal method

   Splitting method

3. Solve, using the null factor law.

SOLVING BY FACTORISATION

Example: Solve for x:

a. $x^2 - 8x = 0$

$x(x-8) = 0$

$x = 0$ or $x - 8 = 0$

$x = 0$ or $x = 8$

b. $x^2 = 7x$

$x^2 - 7x = 0$

$x(x-7) = 0$

$x = 0$ or $x - 7 = 0$

$x = 0$ or $x = 7$

c. $4x^2 = 12x$

$4x^2 - 12x = 0$

$4x(x-3) = 0$

$4x = 0$ or $x - 3 = 0$

$x = 0$ or $x = 3$

d. $x^2 + 2x = 8$

$x^2 + 2x - 8 = 0$

$p = -8$

$s = 2$

$4, -2$

$(x+4)(x-2) = 0$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ or $x = 2$

e. $x^2 - 11x = -24$

$x^2 - 11x + 24 = 0$

$p = 24$

$s = -11$

$-8, -3$

$(x-8)(x-3) = 0$

$x - 8 = 0$ or $x - 3 = 0$

$x = 8$ or $x = 3$

f. $2x^2 + 20x + 50 = 0$

$2(x^2 + 10x + 25) = 0$

$p = 25$

$s = 10$

$5, 5$

$2(x+5)(x+5) = 0$

$x + 5 = 0$

$x = -5$

g. $2x^2 = 3x - 1$ GC

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$ (GC)

$2x - 1 = 0$ or $x - 1 = 0$

$2x = 1$ or $x = 1$

$x = \frac{1}{2}$ or $x = 1$

h. $3x^2 - x = 10$ GC

$3x^2 - x - 10 = 0$

$(3x+5)(x-2) = 0$ (GC)

$3x + 5 = 0$ or $x - 2 = 0$

$3x = -5$ or $x = 2$

$x = \frac{-5}{3}$ or $x = 2$

i. $(x+6)(x-3)=10x$

expand first

$x^2-3x+6x-18=10x$
$x^2+3x-10x-18=0$
$x^2-7x-18=0$

$p=-18$
$s=-7$

$2,-9$

$(x+2)(x-9)=0$

$x+2=0$ or $x-9=0$
$x=-2$ or $x=9$

j. $x(3+x)+7=35$

$3x+x^2+7=35$
$x^2+3x+7-35=0$
$x^2+3x-28=0$

$p=-28$
$s=3$

$7,-4$

$(x+7)(x-4)=0$

$x+7=0$ or $x-4=0$
$x=-7$ or $x=4$

k. $\frac{x-2}{x}=\frac{6+x}{2}$

cross multiply

$2(x-2)=x(6+x)$
$2x-4=6x+x^2$
$-x^2-6x+2x-4=0$
$-x^2-4x-4=0$
$x^2+4x+4=0$

$p=4$
$s=4$

$2,2$

$(x+2)(x+2)=0$

$x+2=0$
$x=-2$

l. $3x+\frac{2}{x}=-7$ GC

$3x^2+2=-7x$
$3x^2+7x+2=0$

$(3x+1)(x+2)=0$ (GC)

$3x+1=0$ or $x+2=0$
$3x=-1$ or $x=-2$
$x=\frac{-1}{3}$ or $x=-2$

Exercise 10C: *page 195*
Questions: *all*

Exercise 10D – Completing the Square

With quadratic equations, you will have noticed all the ones we have dealt with so far have worked out nicely for us. This won't always be the case, so we need a method that will work for us when factorising doesn't.

There are actually two methods that always work:

1. Completing the square
2. Using the quadratic formula.

SOLVING BY COMPLETING THE SQUARE

Consider the equation to solve: $x^2 + 6x - 1 = 0$. We can't use factorisation to help, so we will use the new method. The steps are listed below, let's practice each step with the equation above.

Here are the steps:

$x^2 + 6x - 1 = 0$

1. Rearrange the equation so that $ax^2 + bx = -c$	$x^2 + 6x = 1$
2. a. Halve the number in front of the x **b.** Add the square of this number to **both sides** of the equation	$x^2 + 6x + (3)^2 = 1 + (3)^2$
3. Factorise the LHS of the equation	$(x + 3)^2 = 1 + 9$
4. Rearrange to find x	$(x+3)^2 = 10$ $x + 3 = \pm\sqrt{10}$ $x = -3 \pm \sqrt{10}$

Example: Solve the following by completing the square:

a. $x^2 + 8x = -5$

$x^2 + 8x + (4)^2 = -5 + (4)^2$

$(x+4)^2 = -5 + 16$

$(x+4)^2 = 11$

$x + 4 = \pm\sqrt{11}$

$x = -4 \pm \sqrt{11}$

b. $x^2 - 6x = 13$

$x^2 - 6x + (-3)^2 = 13 + (-3)^2$

$(x-3)^2 = 13 + 9$

$(x-3)^2 = 22$

$x - 3 = \pm\sqrt{22}$

$x = 3 \pm \sqrt{22}$

Example: Solve for x by completing the square:

a. $x^2 - 4x + 6 = 0$

$x^2 - 4x = -6$

$x^2 - 4x + (-2)^2 = -6 + (-2)^2$

$(x - 2)^2 = -6 + 4$

$(x - 2)^2 = -2$

$\therefore$ no solution

* can't square root a negative

b. $3x^2 + 6x - 2 = 0$

$3x^2 + 6x = 2$

$3(x^2 + 2x) = 2$

$x^2 + 2x = \frac{2}{3}$

$x^2 + 2x + (1)^2 = \frac{2}{3} + (1)^2$

$(x + 1)^2 = \frac{2}{3} + 1$

$(x + 1)^2 = \frac{5}{3}$

$x + 1 = \pm\sqrt{5/3}$

$x = -1 \pm \sqrt{5/3}$

Exercise 10D: *page 198*
Questions: *2, 3*

Exercise 10E – The Quadratic Formula

The quadratic formula is the other fail-safe method to solving a quadratic equation.

The rule: If $ax^2 + bx + c = 0$ where $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The rule looks harder than it is. Most students prefer this to completing the square, but you need to know both ways, as you will be instructed to use each method at different times.

Example: Solve for x, by using the quadratic formula, and check using technology.

a. $x^2 - 2x - 2 = 0$

$a = 1 \quad b = -2 \quad c = -2$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1 \times -2)}}{2 \times 1}$$

$$x = \frac{2 \pm \sqrt{4 + 8}}{2}$$

$$x = \frac{2 \pm \sqrt{12}}{2}$$

b. $2x^2 + 3x - 4 = 0$

$a = 2 \quad b = 3 \quad c = -4$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(2 \times -4)}}{2 \times 2}$$

$$x = \frac{-3 \pm \sqrt{9 + 32}}{4}$$

$$x = \frac{-3 \pm \sqrt{41}}{4}$$

c. $x^2 - 6x + 20 = 0$

$a = 1 \quad b = -6 \quad c = 20$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1 \times 20)}}{2 \times 1}$$

$$x = \frac{6 \pm \sqrt{36 - 80}}{2}$$

$$x = \frac{6 \pm \sqrt{-44}}{2}$$

$\therefore$ no solution

d. $\frac{x+4}{x-2} = \frac{5x}{x-1}$

$$(x+4)(x-1) = 5x(x-2)$$

$$x^2 - x + 4x - 4 = 5x^2 - 10x$$

$$x^2 - 5x^2 + 3x + 10x - 4 = 0$$

$$-4x^2 + 13x - 4 = 0$$

$a = -4 \quad b = 13 \quad c = -4$

$$x = \frac{-13 \pm \sqrt{(13)^2 - 4(-4 \times -4)}}{2 \times -4}$$

$$x = \frac{-13 \pm \sqrt{169 - 64}}{-8}$$

$$x = \frac{13 \pm \sqrt{105}}{8}$$

Exercise 10E: *page 200*
Questions: *all*