CHAPTER 4 NOTES – ALGEBRAIC FRACTIONS

Exercise 4A – Evaluating Algebraic Fractions

Example: If $a = 2$, $b = -3$ and $c = -5$, evaluate:

a. $\frac{a-b}{c}$

$= \frac{2-(-3)}{(-5)}$

$= \frac{2+3}{-5}$

$= \frac{5}{-5}$

$= -1$

b. $\frac{a^2-2b}{c-b}$

$= \frac{(2)^2-2(-3)}{(-5)-(-3)}$

$= \frac{4+6}{-5+3}$

$= \frac{10}{-2}$

$= -5$

c. $\frac{a-c-b}{b-a}$

$= \frac{2-(-5)-(-3)}{(-3)-2}$

$= \frac{2+5+3}{-3-2}$

$= \frac{10}{-5}$

$= -2$

Exercise 4A: *page 59*
Questions: *all*

Exercise 4B – Simplifying Algebraic Fractions

When simplifying fractions, keep in mind that you can only cancel when fractions are written in **factored form**. See the examples below.

Example: Simplify, if possible:

a. $\frac{a^2}{2a}$

$= \frac{a \times a}{2 \times a}$

$= \frac{a}{2}$

b. $\frac{6a^2b}{3b}$

$= \frac{6 \times a \times a \times b}{3 \times b}$

$= 2a^2$

c. $\frac{4a}{9b}$

$= \frac{4a}{9b}$

d. $\frac{a+b}{a}$

$= \frac{a+b}{a}$

e. $\frac{(-4b)^2}{2b}$

$= \frac{-4 \times -4 \times b \times b}{2 \times b}$

$= \frac{16 \times b}{2}$

$= 8b$

f. $\frac{18}{3(c-1)}$

$= \frac{18^6}{3(c-1)}$

$= \frac{6}{c-1}$

g. $\frac{3(x+4)}{x+4}$ = $3(x+4)$

h. $\frac{(2x+3)(x+4)}{5(2x+3)}$ = $\frac{x+4}{5}$

i. $\frac{4(x+2)(x-1)}{2(x-1)}$ = $2(x+2)$

Exercise 4B.1: *page 60*
Questions: *all*

FACTORISATION AND SIMPLIFICATION

When Simplifying fractions, keep in mind that you can only cancel when fractions are written in **factored form**. Hmmm… I think I've said this before – this is about to become very important!

Example: Simplify, if possible:

a. $\frac{3a+9}{3}$ = $\frac{3(a+3)}{3}$ = $a+3$

b. $\frac{4a+12}{8}$ = $\frac{4(a+3)}{8}$ = $\frac{a+3}{2}$

c. $\frac{3x-6}{2}$ = $\frac{3(x-2)}{2}$

d. $\frac{ab+ac}{b+c}$ = $\frac{a(b+c)}{b+c}$ = a

e. $\frac{6x^2-6xy}{3x-3y}$ = $\frac{6x(x-y)}{3(x-y)}$ = $2x$

f. $\frac{8x+8y}{6x^3+4x^2y}$ = $\frac{8(x+y)}{2x^2(3x+2y)}$ = $\frac{4(x+y)}{x^2(3x+2y)}$

Some questions will look like we can't simplify them, but we can often be a bit savvy and whip -1 out as a common factor - voilà! Now we can simplify.

Example: Simplify, if possible:

a. $\frac{6a-6b}{b-a}$ = $\frac{6(a-b)}{-1(a-b)}$ = -6

b. $\frac{4a-8b}{6b-3a}$ = $\frac{4(a-2b)}{-3(a-2b)}$ = $\frac{-4}{3}$

c. $\frac{xy^2-xy}{1-y}$ = $\frac{xy(y-1)}{-1(y-1)}$ = $-xy$

d. $\frac{x^2-9}{x+3}$

$= \frac{(x+3)(x-3)}{x+3}$

$= x-3$

e. $\frac{a^2-b^2}{2a+2b}$

$= \frac{(a+b)(a-b)}{2(a+b)}$

$= \frac{a-b}{2}$

f. $\frac{x^2-1}{x^2+3x+2}$

$= \frac{(x+1)(x-1)}{(x+2)(x+1)}$

$= \frac{x-1}{x+2}$

p = 2
s = 3
1, 2

Exercise 4B.2: *page 62*
Questions: *all*

Exercise 4C – Multiplying and Dividing Fractions

When we **multiply fractions** together, we just multiply the top line by the top line, and multiply the bottom line by the bottom line.

It's easier if we cancel before multiplying. If you want to cancel something, make sure the problem is in simplest form, and check that the thing you want to cancel is on the top and bottom line of a fraction.

They do not have to be directly above and below each other!

Example: Simplify:

a. $\frac{3}{m} \times \frac{m}{6}$

$= \frac{1}{2}$

b. $\frac{3}{m} \times m^2$

$= \frac{3}{m} \times \frac{m^2}{1}$

$= 3m$

c. $\left(\frac{a}{b}\right)^2$

$= \frac{a^2}{b^2}$

When we **divide fractions**, the first step is to flip the second fraction, and then multiply. Do not do any cancelling before flipping and changing the division sign to a multiplication sign. You will do yourself a mischief if you try and cancel first!

Example: Simplify:

a. $\frac{4}{n} \div \frac{2}{n^2}$

$= \frac{2}{4} \times \frac{n^2}{2}$

$= 2n$

b. $\frac{3}{a} \div 2$

$= \frac{3}{a} \times \frac{1}{2}$

$= \frac{3}{2a}$

c. $\frac{1}{3n} \div 9n^2$

$= \frac{1}{3n} \times \frac{1}{9n^2}$

$= \frac{1}{27n^3}$

Exercise 4C: *page 66*
Questions: *1, 2*

Exercise 4D – Adding and Subtracting Algebraic Fractions

To add and subtract algebraic fractions, we need a common denominator.

Example: Simplify, by writing as a single fraction:

a. $\frac{x}{2} + \frac{3x}{4}$ ($\times 2$, $\times 1$)

$= \frac{2x}{4} + \frac{3x}{4}$

$= \frac{5x}{4}$

b. $\frac{a}{3} - \frac{2a}{3}$

$= \frac{-a}{3}$

c. $\frac{x}{6} + \frac{(x-2)}{3}$ ($\times 2$)

$= \frac{x}{6} + \frac{2(x-2)}{6}$

$= \frac{x}{6} + \frac{2x-4}{6}$

$= \frac{3x-4}{6}$

d. $\frac{(x+1)}{2}$ ($\times 3$) $- \frac{(x-2)}{3}$ ($\times 2$)

$= \frac{3(x+1)}{6} - \frac{2(x-2)}{6}$

$= \frac{3x+3}{6} - \frac{2x+4}{6}$

$= \frac{x+7}{6}$

Exercise 4D: *page 67*
Questions: *1, 5*