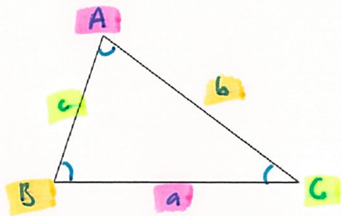
CHAPTER 6 NOTES – NON-RIGHT ANGLED TRIANGLE TRIGONOMETRY

Exercise 6A – Areas of Triangles

We are used to finding the area of a triangle by using the formula $A = \frac{1}{2}b \times h$, but we need to know the perpendicular height to be able to use this formula. If we're not given the perpendicular height, don't fear, we now have a new formula up our sleeve!

Before we get into the new formula, we are first going to get into the habit of labelling our triangles, and we use a specific way to do this. We will stick with labelling the sides a, b and c – and use lower case letters. Then, on the angle opposite each side, label it A, B and C. Let's practise!

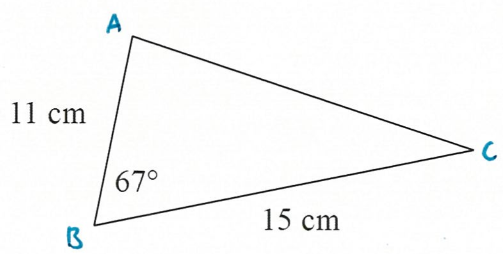


It will never matter which side you call a particular letter, as long as you have its capital letter on the angle opposite it.

Now we can find the area of any triangle, using the formula $A = \frac{1}{2}ab \sin C$.

Depending on how you have labelled your triangle, you could also use $A = \frac{1}{2}bc \sin A$ or $A = \frac{1}{2}ac \sin B$. It is all the same thing. Just note that for us to have the info we need, we need to know two sides, and the included angle.

Example: Find the area of the triangle.



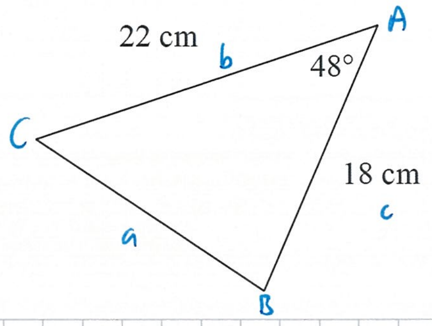
$$\text{Area} = \frac{1}{2} ac \sin B$$

$$= \frac{1}{2} \times 15 \times 11 \times \sin(67)$$

DEGREES

$$\approx 75.9 \text{ cm}^2$$

Example: Find the area of the triangle:



$$\text{Area} = \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} \times 22 \times 18 \times \sin 48$$

$$\approx 147.14 \text{ cm}^2$$

Example: A triangle has two sides with lengths $10\ cm$ and $11\ cm$, and an area of $50\ cm^2$. Determine the possible measures of the included angle. Give your answers accurate to 1 decimal place.

let included angle be θ.

then $\text{Area} = \frac{1}{2} \times 10 \times 11 \times \sin\theta$

$50 = 55 \sin\theta$

$\sin\theta = \frac{50}{55}$

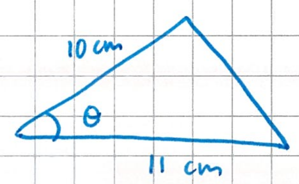
$\theta = \sin^{-1}\left(\frac{50}{55}\right)$

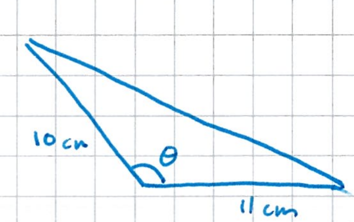
$\theta \approx 65.4^\circ$ OR $180^\circ - 65.4^\circ$

$\theta \approx 65.4^\circ$ OR 114.6°

$\therefore$ the two possible angles are 65.4° or 114.6°.

TWO OPTIONS

① 

② 

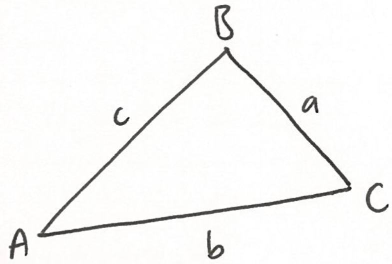
***Exercise 6A:** page 148*
***Questions:** all*

Exercise 6B – The Cosine Rule

We can find angles and lengths of sides of a triangle using the **cosine rule**. Just like with the area formula, it will seem like there are three versions, but remember they are the same thing, but there are just three versions depending on how we label our triangle.

To find lengths:

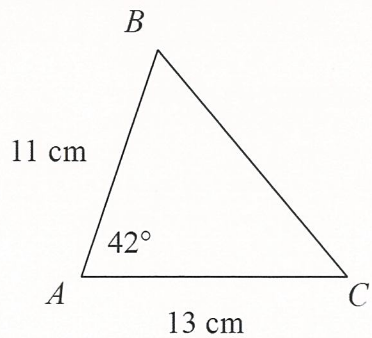
$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$



To find angles:

$\cos A = \frac{b^2+c^2-a^2}{2bc}$
$\cos B = \frac{a^2+c^2-b^2}{2ac}$
$\cos C = \frac{a^2+b^2-c^2}{2ab}$

Example: Find, correct to 2 decimal places, the length of BC.



$BC^2 = 11^2 + 13^2 - 2\times 11\times 13\times\cos 42$ (cosine rule)

$BC = \sqrt{11^2 + 13^2 - 2\times 11\times 13\cos 42}$

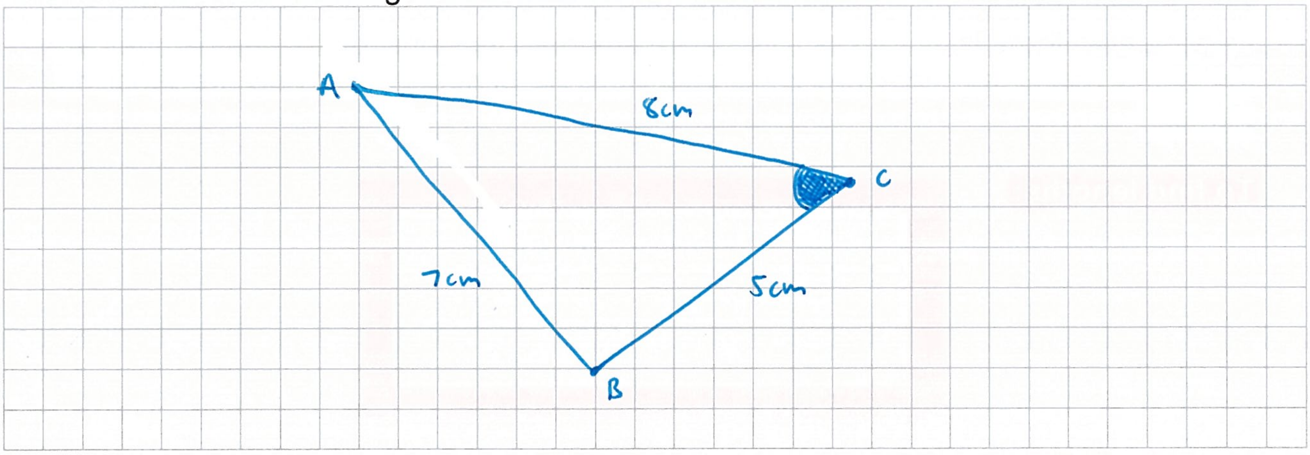
$BC \approx 8.80$

DEGREES

$\therefore$ [BC] is approximately 8.8 cm long

Example: In triangle ABC, $AB = 7\ cm, BC = 5\ cm$, and $CA = 8\ cm$.

a. Draw the triangle.



b. Find the measure of angle C.

$$\cos C = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$ (cosine rule)

$$C = \cos^{-1}\left(\frac{40}{80}\right)$$

$$C = 60^\circ$$

$\therefore$ angle C is 60°.

c. Find the exact area of triangle ABC.

$$A = \frac{1}{2} \times 8 \times 5 \times \sin 60$$

$$A = 20 \times \frac{\sqrt{3}}{2}$$

$$A = 10\sqrt{3}\ cm^2$$

Exercise 6B: *page 151*
Questions: *all*

Exercise 6C – The Sine Rule

The **sine rule** is another tool we can use to find lengths and angles of triangles. Whilst it looks like the formula requires a whole lot of information, we only use the part of the formula which is useful for us.

To find lengths:

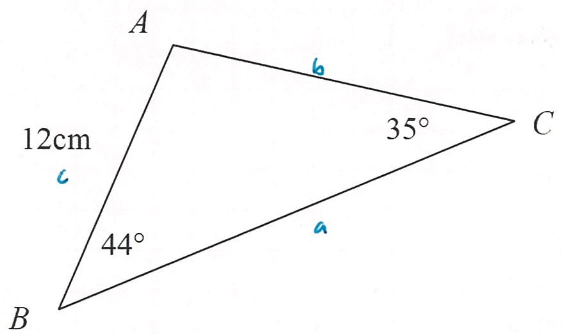
$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

To find angles:

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

We will first practise finding lengths, as there is a little tricky bit to finding angles which we will need to deal with – so we'll save that for later.

Example: Find the length of AC, correct to 2 decimal places.



$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 44} = \frac{12}{\sin 35}$$

$$b = \frac{12 \times \sin 44}{\sin 35}$$

$$b = 14.5 \text{ cm}$$

Exercise 6C.1
Question 2a:

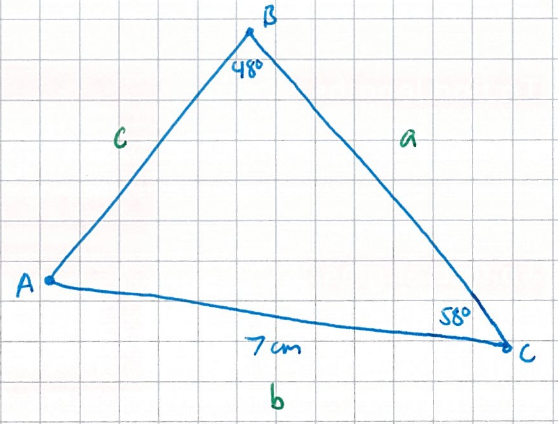
For triangle ABC, find all unknown sides and angles if $AC = 7\ cm$, $angle\ C = 58°$ and $angle\ B = 48°$.

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$ (sine rule)

$$\frac{c}{\sin 58} = \frac{7}{\sin 48}$$

$$c = \frac{7 \times \sin 58}{\sin 48}$$

$$c = 7.99\ cm$$



Angle $A = 180 - (48 + 58)$ (angles in a triangle)

$$A = 180 - 106$$

$$A = 74°$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$ (sine rule)

$$\frac{a}{\sin 74} = \frac{7}{\sin 48}$$

$$a = \frac{7 \times \sin 74}{\sin 48}$$

$$a = 9.05\ cm$$

***Exercise 6C.1:** page 154*
***Questions:** 1, 2b, c, 3, 4*

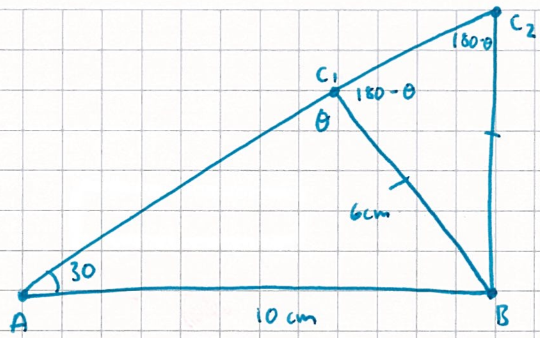
We will now use the formula to find angles in triangles. Please note that when we are finding angles, we are not normally given enough information to define a particular triangle – **there are often two triangles which may be** possible.

We need to consider this in our answer. I will explain this further on the board – room to copy below.

CASE 1

Two possible angles for C.

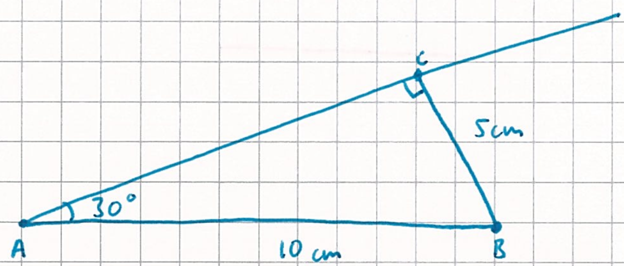
Two triangles



CASE 2

One possible (acute) angle for C.

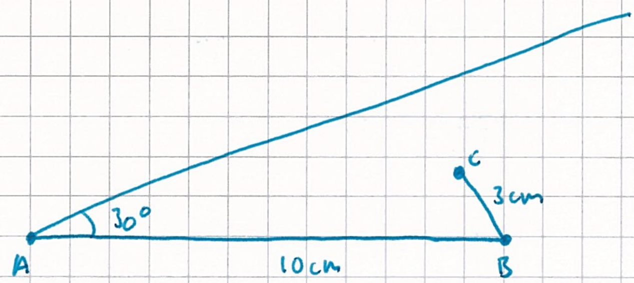
One triangle



CASE 3

No possible angle for C.

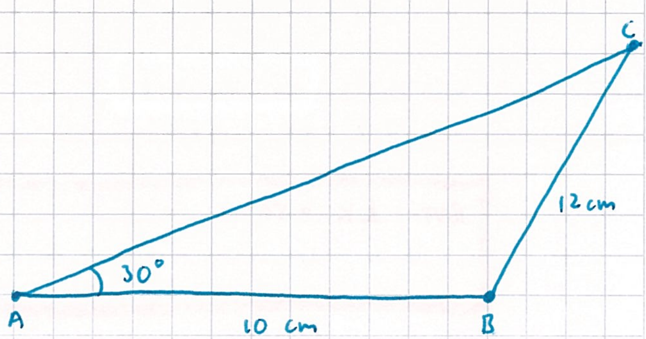
No triangle



CASE 4

One possible (obtuse) angle for C.

One triangle



Example: Find the measure of angle C in triangle ABC if $AC = 7\ cm, AB = 11\ cm$ and angle B measures 25˚

$\frac{\sin C}{c} = \frac{\sin B}{b}$ (sine rule)

$\frac{\sin C}{11} = \frac{\sin 25}{7}$

$\sin C = \frac{11 \times \sin 25}{7}$

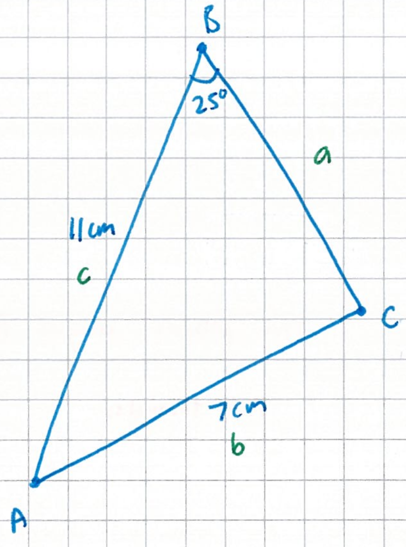
$C = \sin^{-1}\left(\frac{11 \times \sin 25}{7}\right)$

$C \approx 41.6°$ $\boxed{\text{OR} \quad 180 - 41.6°}$

$C = 41.6°$ OR $138.4°$

If $\triangle ABC$ is acute, $C = 41.6°$

if $\triangle ABC$ is obtuse, $C = 138.4°$



Example: Find the measure of angle L in triangle KLM given that angle K measures $56°, LM = 16.8\ m$, and $KM = 13.5\ m$.

$\frac{\sin L}{l} = \frac{\sin K}{k}$ (sine rule)

$\frac{\sin L}{13.5} = \frac{\sin 56}{16.8}$

$\sin L = \frac{13.5 \times \sin 56}{16.8}$

$L = \sin^{-1}\left(\frac{13.5 \times \sin 56}{16.8}\right)$

$L \approx 41.8°$ $\boxed{\text{OR} \quad 180 - 41.8°}$

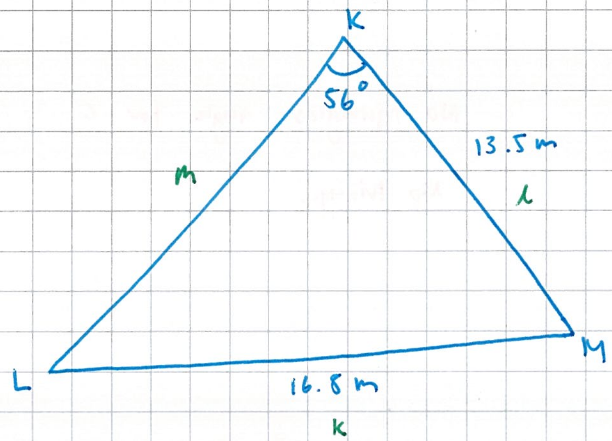
$L = 41.8°$ OR $138.2°$

BUT $\angle K = 56°$

If $L = 138.2°$, $138.2 + 56 > 180$ (angles in triangle)

$\therefore L \neq 138.2°$

$\therefore$ Angle L is $41.8°$



***Exercise 6C.2:** page 157*
***Questions:** all*

Exercise 6D – Problem Solving

When we are given problem solving questions – the following steps will help!

1. Draw a triangle.
2. Label the triangle and find any angle that you can.
3. Decide which rule to use:

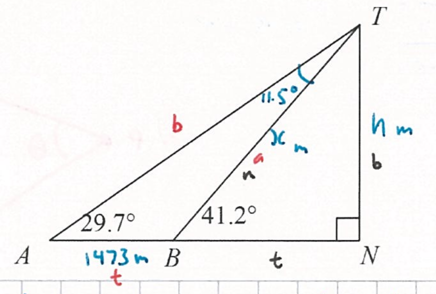
   a. Use the **cosine rule** to find **length** if given 2 sides and an included angle.
   b. Use the **cosine rule** to find an **angle** if given three sides.

   c. Use the **sine rule** to find a **length** if given one side and 2 angles.
   d. Use the **sine rule** to find an **angle** if given 2 sides and a non-included angle.

   REMEMBER TO CHECK FOR TWO POSSIBLE ANSWERS!!!

4. Answer the question.
5. Pat yourself on the back ☺.

Example: The angles of elevation to the top of a mountain are measured from two beacons A and B at sea. The measurements are shown on the diagram. If the beacons are $1473\ m$ apart, how high is the mountain?



let BT be x m and NT be h m

$\therefore \angle ATB = 180 - (29.7 + (180 - 41.2))$ (angles on a line & in a triangle)
$= 11.5°$

$\therefore$ $\boxed{\text{In } \triangle ABT}$: $\dfrac{x}{\sin 29.7} = \dfrac{1473}{\sin 11.5}$ (sine rule)

$x = \dfrac{1473 \times \sin 29.7}{\sin 11.5}$

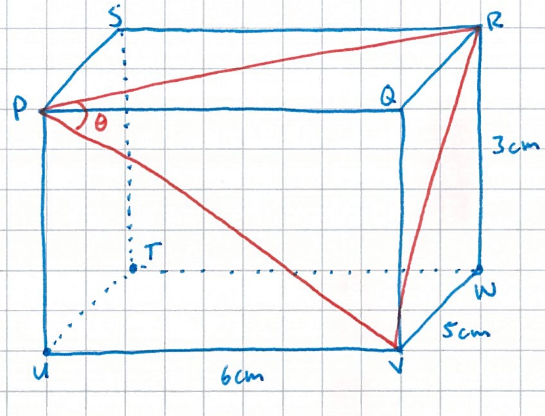
$x = 3660.62\ m$

$\boxed{\text{In } \triangle BNT}$: $\sin 41.2 = \dfrac{h}{x}$

$\therefore h = 3660.62 \times \sin 41.2$
$h = 2410\ m$

$\therefore$ The mountain is 2410 m high.

Example: Find the measure of RPV in the 3-D shape below (drawn together).



* Find lengths of red triangle *

In ΔRVW

$$RV = \sqrt{5^2 + 3^2}$$
$$= \sqrt{34} \text{ cm}$$
(pyth)

In ΔPUV

$$PV = \sqrt{6^2 + 3^2}$$
$$= \sqrt{45} \text{ cm}$$
(pyth)

In ΔPQR

$$PR = \sqrt{6^2 + 5^2}$$
$$= \sqrt{61} \text{ cm}$$
(pyth)

* Find $\angle RPV$ *

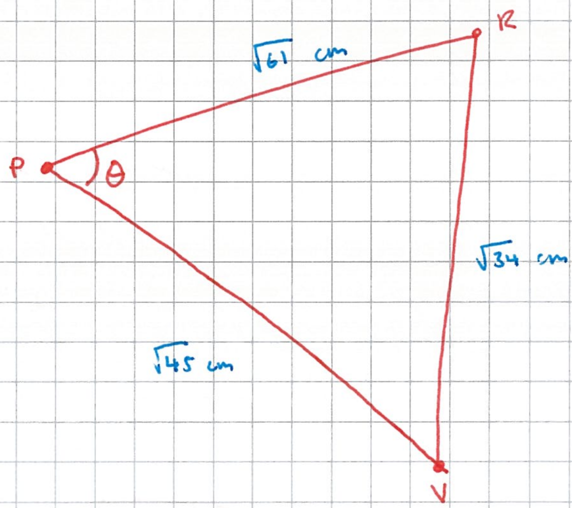
$$\cos\theta = \frac{\sqrt{61}^2 + \sqrt{45}^2 - \sqrt{34}^2}{2 \times \sqrt{61} \times \sqrt{45}}$$
(cosine rule)

$$\cos\theta = \frac{61 + 45 - 34}{2\sqrt{61}\sqrt{45}}$$

$$\cos\theta = \frac{72}{2\sqrt{61}\sqrt{45}}$$

$$\theta = \cos^{-1}\left(\frac{36}{\sqrt{61}\sqrt{45}}\right)$$

$$\theta = 46.6^\circ$$



$\therefore \angle RPV = 46.6^\circ$

Exercise 6D: *page 159*
Questions: *all*