South Australian Certificate of Education
LEARNING AT THE PACE OF CHANGE

Government of South Australia

Mathematics

2021 Subject Outline | Stage 1

For teaching

In Australian and SACE International schools from January 2021 to December 2021

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INTRODUCTION

SUBJECT DESCRIPTION

Stage 1 Mathematics is a 10-credit subject or a 20-credit subject.

Mathematics develops an increasingly complex and sophisticated understanding of calculus, statistics, mathematical arguments, and proofs, and using mathematical models. By using functions, their derivatives, and integrals, and by mathematically modelling physical processes, students develop a deep understanding of the physical world through a sound knowledge of relationships involving rates of change. Students use statistics to describe and analyse phenomena that involve uncertainty and variation.

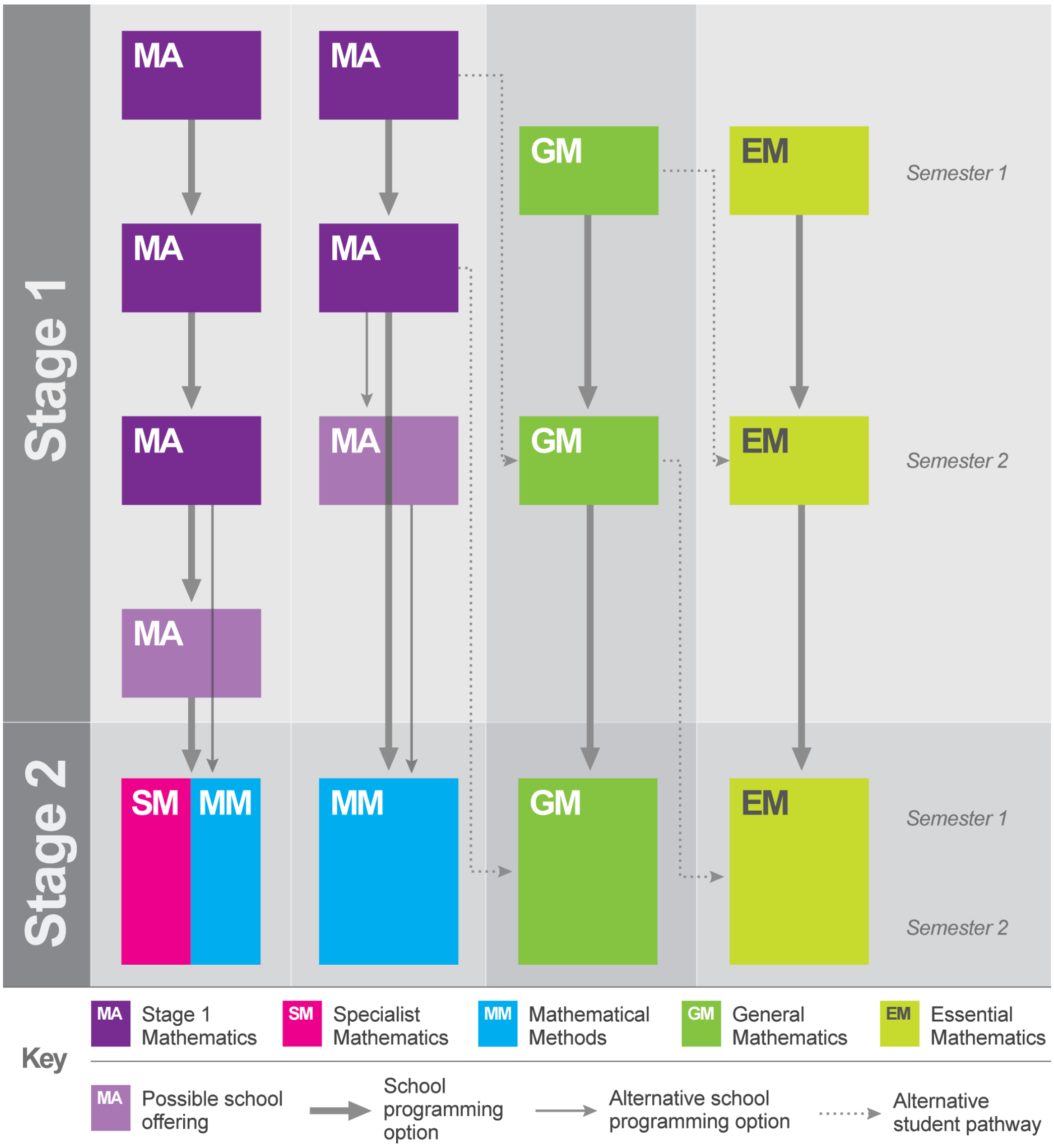
Stage 1 Mathematics provides the foundation for further study in mathematics in Stage 2 Mathematical Methods and Stage 2 Specialist Mathematics.

Stage 2 Mathematical Methods can lead to tertiary studies of economics, computer sciences, and the sciences. It prepares students for courses and careers that may involve the use of statistics, such as health or social sciences.

Stage 2 Specialist Mathematics can be a pathway to mathematical sciences, engineering, space science, and laser physics. Specialist Mathematics is designed to be studied in conjunction with Mathematical Methods.

MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might study at Stage 1 and Stage 2.



Notes:

Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum per se is not a prerequisite for the study of Stage 1 Mathematics. The essential aspects of 10A are included in the relevant topics. See programming notes on page 8 for information about selection of topics and subtopics for Stage 1 Mathematics.

Stage 2 Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:

- literacy
- numeracy
- information and communication technology (ICT) capability
- critical and creative thinking
- personal and social capability
- ethical understanding
- intercultural understanding.

Literacy

In this subject students develop their literacy capability by, for example:

- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphical, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use mathematical skills, concepts, and technologies in a range of contexts that can be applied to:

- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical, and algebraic representations, and other mathematical models.

Information and communication technology (ICT) capability

In this subject students develop their information and communication technology capability by, for example:

- understanding the role of electronic technology in the study of mathematics
- making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and in processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant to study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice, and application; for example, to use data, address problems, and operate systems in particular situations.

Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- developing mathematical reasoning skills to think logically and make sense of the world
- understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

Ethical understanding

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- examining critically ways in which the media present particular perspectives
- sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols that have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 Mathematics with a C grade or better, or 20 credits of Stage 2 Mathematical Methods or Stage 2 Specialist Mathematics with a C– grade or better, will meet the numeracy requirement of the SACE.

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through their learning in Stage 1 Mathematics.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions, solving problems, applying models, and making and testing conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Mathematics at Stage 1 builds on the mathematical knowledge, understanding, and skills that students have developed in Number and Algebra, Measurement and Geometry, and Statistics and Probability during Year 10.

Stage 1 Mathematics is organised into topics that broaden students' mathematical experience, and provide a variety of contexts for incorporating mathematical arguments and problem-solving. The topics provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, and level of sophistication and abstraction.

Key concepts from 10A Mathematics in the Australian Curriculum required for the study of Stage 1 Mathematics, Stage 2 Mathematical Methods, and Stage 2 Specialist Mathematics have been incorporated into the relevant topics.

Stage 1 Mathematics consists of the following twelve topics:

- Topic 1: Functions and graphs
- Topic 2: Polynomials
- Topic 3: Trigonometry
- Topic 4: Counting and statistics
- Topic 5: Growth and decay
- Topic 6: Introduction to differential calculus

- Topic 7: Arithmetic and geometric sequences and series
- Topic 8: Geometry
- Topic 9: Vectors in the plane
- Topic 10: Further trigonometry
- Topic 11: Matrices
- Topic 12: Real and complex numbers.

Programming

Students who want to undertake Stage 2 Mathematical Methods should study at least 20 credits of Stage 1 Mathematics. This may be two 10-credit subjects or one 20-credit subject.

Students who want to undertake Stage 2 Specialist Mathematics should study at least 10 additional credits of Stage 1 Mathematics.

Programs for a 10-credit subject must be made up of a selection of subtopics from at least three topics. Topics can be studied in their entirety or in part.

Programs for a 20-credit subject must be made up of a selection of subtopics from at least six topics. Topics can be studied in their entirety or in part.

As a guide, Topics 1 to 6 prepare students for the study of Stage 2 Mathematical Methods and Topics 7 to 12 prepare students for the study of Stage 2 Specialist Mathematics.

Note that the topics have not been designed to be of equivalent length. It is anticipated that some topics will need a greater allocation of time than others.

The topics selected can be sequenced and structured to suit individual cohorts of students. The suggested order of the topics provided in the list is a guide only. Each topic consists of a number of subtopics. These are presented in the subject outline in two columns, as a series of key questions and key concepts, side by side with considerations for developing teaching and learning strategies.

The key questions and key concepts cover the content for teaching, learning, and assessment in this subject. The considerations for developing teaching and learning strategies are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions, students deepen their understanding of concepts and processes that relate to the mathematical models required to address the problems posed.

The considerations for developing teaching and learning strategies present problems and guidelines for sequencing the development of the key questions and key concepts. They also give an indication of the depth of treatment and emphases required.

Students use electronic technology, where appropriate, to enable complex problems to be solved efficiently.

Topic 1: Functions and graphs

A key aspect of mathematics is to model real-world situations. To do this effectively, skills enabling students to work algebraically with functions and relations are essential. Using a system of coordinates allows numerical descriptions of different situations to be described using a function or a relation.

Linear functions are suitable for exploring relationships between variables that show a constant rate of change, such as simple interest and power usage. Inverse relationships are used to study contexts in which one variable increases as another variable decreases, such as the effect of increasing the number of workers has on the completion time of a building job. Circular relationships can be used to model, for example, the locations of earthquakes and the coverage from mobile phone towers.

In combination with Topic 2: Polynomials, the concepts and techniques covered in this topic develop the understanding of functions and relations, and algebraic skills leading to the study of calculus.

The emphasis in this topic is on describing, sketching, interpreting, and discussing the behaviour of graphs that arise from everyday situations. Students focus on describing and explaining the characteristics and behaviour of a graph in relation to the situation being modelled.

The investigation of links between the algebraic and graphical representations of functions relies on the use of technology for the production of graphs of mathematical functions. Students test their conjectures using many examples, without plotting graphs themselves.

Subtopic 1.1: Lines and linear relationships

Key questions and key concepts	Considerations for developing teaching and learning strategies

How can all the points on a straight line be described mathematically?
• The equation of a straight line
  ◆ from two points
  ◆ from a slope and a point
  ◆ parallel and perpendicular to a given line through some other point
	In the context of a city environment, straight lines occur as roads, storm water drains, gas pipelines, and so on. The location of such infrastructure can be described as passing through two distinct places or as originating at a particular point and travelling in a specified way.
	
	From this concept comes the equation of a straight line, found from being given either two points or one point and a slope. Equations of lines that are parallel or perpendicular to given lines can be found.
	
	It is possible to find the distance between two points (the length of the road or storm water drain) or the middle point between two points.

What are the features of the graph of a linear function $y = mx + c$? • Slope (m) as a rate of growth • y-intercept (c)	When slope is related to the constant adder, its role as a rate of growth becomes clear. An example is the cost per kilometre of a taxi journey. Depending on the context, the y-intercept can be interpreted to be the initial condition, such as the flag fall of a taxi ride.
How do you work out the formula for a linear relationship, given some data or a description of a situation? • Slope as a rate of growth • Interpretation of the intercepts	The functions used are drawn from everyday contexts (e.g. simple interest, water rates, conversion graphs, telephone charges). When slope is related to a constant adder, its role as a rate becomes clear. The axis intercepts are interpreted in context.
How can the point where two lines intersect be found? • Solve simultaneous linear equations, graphically and algebraically • Find the points of intersection between two coincident straight lines	Where will two straight roads intersect? At what location will a proposed pipeline have to pass under a road? How is it possible to tell that the railway and the road are parallel or perpendicular? These questions involve the solution of a linear equation or a pair of simultaneous linear equations and the interpretation of that solution.

Subtopic 1.2: Inverse proportion

Key questions and key concepts	Considerations for developing teaching and learning strategies
What kind of mathematical relationship describes the situation in which one variable decreases as the other increases?	The concept of the inverse proportion embodied in these relationships is studied in the context of the provision of services. For example: How does the time for service vary as the number of providers increases?
What are the features of the graph of $y = \frac{1}{x}$?	How are the length and width of an envelope of a standard weight (and hence area) related? The equation and its graph in the Cartesian plane show the relationship between the two changing variables.
These graphs feature horizontal and vertical asymptotes.	Students investigate translations of the basic hyperbola in the form $y = \frac{a}{(x-c)}$.

Subtopic 1.3: Relations

Key questions and key concepts	Considerations for developing teaching and learning strategies
What kind of equation describes a circle of which you know the radius and the location of its centre? • Equations of circles in both centre/radius and expanded form	Consider the mathematical description of the region covered by a mobile telephone tower or a pizza shop that delivers within a certain distance. Converting the equation of a circle from expanded form provides practise on 'completing the square'. Features of $y^2 = x$ include its parabolic shape and its axis of symmetry — an example of a relation that is not a function. The features of the graph of $y^2 = x$ and $y = x^{\frac{1}{2}}$ can also be considered.

Subtopic 1.4: Functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is meant by the term 'function'? The concept of a function (and the concept of the graph of a function) • Domain and range • The use of function notation • Dependent and independent variables	This includes the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable in terms of another.
	The graph of a function *f* is all of the points (x, y) on a Cartesian plane where x is in the domain of *f* and $y = f(x)$. The values of y define the range. The use of function notation is developed, including finding the value of functions and function of functions. The concept of dependent and independent variables is considered in different contexts leading to the labelling of the axes. If we write $y = f(x)$, we say that x is the independent variable and y is the dependent variable.
What is the distinction between functions and relations? Recognise the distinction between functions and relations.	Consider the relations in Subtopic 1.3 and other examples of relations and determine whether or not they are functions. Using the vertical line test is a practical way of determining if a relation is a function.

Topic 2: Polynomials

Building on from the skills and understanding that students have developed in Topic 1: Functions and graphs, this topic involves further modelling of real-world situations. Polynomial functions are used for exploring relationships that are more complex than linear models.

As students gain a sound understanding of the graphical behaviour of these functions, they develop their skills in the algebraic manipulation of polynomials. The links between these two concepts are strengthened by the use of electronic technology.

Subtopic 2.1: Quadratic relationships

Key questions and key concepts	Considerations for developing teaching and learning strategies
Where do quadratic relationships arise in everyday situations?	Students construct quadratic relationships from given situations and examine existing models from a range of contexts. Possible contexts include areas of rectangles with fixed perimeters (including golden rectangles), business applications (profit functions), elastic collisions, and projectile paths, such as shooting netball goals, voltage, and electrical power. One example is to throw a ball straight up, from 1 m above the ground with a velocity of 4 m s^{-1}. Ignoring air resistance, the height (h) in metres is $h=1+4t-5t^2$ where t is the time in seconds. (The $-5t^2$ is an approximation for $\frac{1}{2}at^2$, where $a=-9.80\,\mathrm{m\,s^{-2}}$).
What are the features of the graph of $y=x^2$ and how are the graphs of $y=a(x-b)^2+c$ and $y=a(x-\alpha)(x-\beta)$ related?	Features of the graphs of quadratic functions include the parabolic nature, intercepts, turning points, and axes of symmetry. Explore graphs using technology or by drawing up tables of values.
How can quadratic expressions be rearranged algebraically so that more can be learnt about their usefulness in solving problems? • Factorisation of quadratics of the form ax^2+bx+c and hence determine zeros • The quadratic formula to determine zeros • Completing the square and hence finding turning points • The discriminant and its significance for the number and nature of the zeros of a quadratic equation and the graph of a quadratic function • Using technology	Beginning with the simpler models, students translate between the different algebraic forms of a quadratic expression. While they are doing this, there is an emphasis on the equivalence of the algebraic expressions and the information that each form provides about the model and its graph. Students use appropriate technology to examine approximate and exact solutions to quadratic equations. Students become aware of the limitations of the different techniques so that they can make an appropriate choice when looking for a solution to a quadratic equation. Positive and negative definite functions can be discussed as well as those with identical zeros. For the 'throwing the ball' example $h=1+4t-5t^2$, the maximum height and length of the time in the air can be calculated.

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is the relationship between the solutions of a quadratic equation, the algebraic representation of the associated quadratic function, and its graph? • The sum and product of the real zeros of a quadratic equation, and the associated algebra of surds	These relationships are crucial to the next step in analysing various situations: that is, determining (algebraically) a quadratic model to fit given data.
How can knowledge about quadratic functions be used to determine these relationships from data? • Deducing quadratic models from the zeros and one other piece of data (e.g. another point), using suitable techniques and/or technologies • Understand the role of the discriminant	Students compare relationships determined algebraically and those found using some form of technology. Graphing technology allows a student to fit a relationship by trial and error, using the graph to determine how well it fits and also to test the uniqueness of the result. Once an appropriate quadratic relationship has been found (by whatever method), it is used to answer questions or make predictions about the situation being modelled.

Subtopic 2.2: Cubic and quartic polynomials

Key questions and key concepts	Considerations for developing teaching and learning strategies
What kinds of models have a cubic relationship? What language is used to describe the nature of the polynomial? • Leading coefficient • Degree	Students explore relationships between volume and linear measure (e.g. the volume of a box created by cutting squares from the corners of a rectangular piece of card). Other contexts where cubic relationships arise are solubility of chemicals versus temperature, and wind speed versus power output from a wind generator.
What kinds of behaviour can be expected from the graph of a cubic function? • $y = x^3$ • $y = a(x-b)^3 + c$ • $y = a(x-\alpha)(x-\beta)(x-\gamma)$	Students use technology to investigate the graphs of a range of cubic functions with a view to identifying the shape and the number of zeros. For example, what is the effect on the graph if the leading coefficient $a \neq 1$?
What algebraic forms can a cubic expression take? • Cubics can be written as a product of a linear and a quadratic factor or as a product of three linear factors • What is the significance of these forms for the shape and number of zeros of the graph? • Cubic equations can be solved algebraically and by using technology	Students use multiplication to verify the equivalence of factorised and expanded forms of cubic polynomials. Given one linear factor of a real cubic, using a quadratic with unknown coefficients, students find the other factors by inspection or by equating coefficients. By factorising a cubic where a linear factor is known, students can solve cubic equations algebraically. Students can also use technology to solve cubic equations. Students understand what happens when $x \to \pm\infty$. The examples can be extended to look at polynomials of degree greater than 3.
What kinds of models have a fourth power (or quartic) relationship?	As with the quadratic and cubic functions, students examine models that give rise to quartic functions (e.g. temperature and radiated heat, frequency, and scattering of light) but realise that the study of fourth-degree polynomials is a logical extension of work already done with the simpler functions.
Extending from the quadratic and cubic functions — what kinds of behaviour can be expected from the graphs of quartic functions?	The ideas developed in this subtopic are treated as part of a mathematical modelling process of investigation that begins with a simple case and gradually gains in complexity.

Topic 3: Trigonometry

The study of trigonometry enables students to expand their mathematical modelling into contexts such as construction, design, navigation, and surveying by using periodic functions. The variation in demand for electricity throughout the day, the seasonal variations in climate, and the cycles within the economy are examples of periodic phenomena. Understanding how to model these phenomena, students predict trends in them.

Students extend their understanding of trigonometry into non–right-angled triangles. They learn about one particular family of periodic functions with the introduction of the basic trigonometric functions, beginning with a consideration of the unit circle, using degrees. Radian measure of angles is introduced, the graphs of the trigonometric functions are examined, and their applications in a range of settings are explored. More complex trigonometric functions are explored in Topic 10.

Subtopic 3.1: Cosine and sine rules

Key questions and key concepts	Considerations for developing teaching and learning strategies
What tools are there for solving problems involving right-angled triangles? • Pythagoras' theorem • Trigonometric ratios	Briefly consider right-angled triangle problems in context and with practical activities where appropriate; for example: • Finding the height of an object, using a clinometer • Finding the angle of inclination of the sun • Determining whether or not a volleyball court is truly rectangular • Calculating the length of ladder to reach an otherwise inaccessible spot • Calculating the length of props needed to raise a shed wall into a vertical position.
How is it possible to solve problems in which the triangles involved are not right-angled?	Use of tools to deal with non-right triangles can be emphasised by posing problems in contexts such as surveying, building, navigation, and design. Students consider how they would find the answers to these problems, using the skills they have learnt. Discuss the validity and/or shortcomings of methods (e.g. scale drawing and trial and error).
The cosine rule • Find the length of the third side when two sides and the included angle are known • Find the measure of an angle when the three sides are known	Consider the derivation of the cosine rule, using Pythagoras' theorem. Recognise that the cosine rule is a 'generalised' version of Pythagoras' theorem with a 'correction factor' for angles that are larger or smaller than $90°$. Examples using the cosine rule include the solution of contextual problems drawn from recreation and industry for an unknown side or angle.
The sine rule • Find the measure of an unknown angle when two sides and the non-included angle are known • Find the length of one of the unknown sides where two angles and one side are known	Justification of the sine rule by direct measurement is useful.
• Are there now sufficient tools to solve any problem involving the angles and lengths of sides of triangles?	Examples using the sine and cosine rule include the solution of contextual problems drawn from recreation and industry for an unknown side or angle.

Key questions and key concepts

- How is it possible to find the area of a non-right triangle if the perpendicular to a side cannot be measured easily or accurately?

Considerations for developing teaching and learning strategies

Derivation of the formula $A=\frac{1}{2}ab.\sin C,$ using right triangles.

This formula can be used to establish the sine rule.

Subtopic 3.2: Circular measure and radian measure

Key questions and key concepts	Considerations for developing teaching and learning strategies
What do the graphs of $\cos\theta$ and $\sin\theta$ look like?	As an introduction to sketching trigonometric functions, draw the graphs of $\cos\theta$ and $\sin\theta$ using degrees.
What is the link between the unit circle and $\cos\theta$, $\sin\theta$, and $\tan\theta$ in degrees? Understand the unit circle definition of $\cos\theta$, $\sin\theta$, and $\tan\theta$, and periodicity using degrees.	Use technology to show this link.
Can angles be measured in different units? • Define the radian measure of an angle as the length of the arc that subtends this angle at the centre of a unit circle • Apply the relationship to convert between radian and degree measure	Data generated by measuring the height of the fixed point on a bicycle wheel as it rolls can be standardised by converting the distance travelled into radius units (radians) and the height above the ground into the height above or below the axle.
Calculate lengths of arcs and areas of sectors of circle.	This is an introduction to the concept of radian measure of angles and the 'standardised' unit circle as a frame of reference for considering all situations that involve circular motion.

Subtopic 3.3 Trigonometric functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is the link between the unit circle and $\cos\theta$ and $\sin\theta$ in radians?	Use software to show this link.
• Understanding the unit circle definition of $\cos\theta$ and $\sin\theta$ and periodicity using radians	Determine the exact values of $\cos\theta$ and $\sin\theta$ for integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ using either unit circle or graphs.
What function best describes the horizontal and vertical position of a point moving round a unit circle?	The sine function can be seen as describing the vertical motion of the point.
• The functions $y=\sin x$ and $y=\cos x$	The cosine function can be seen as describing the horizontal motion of the same point. The relationship between the graphs of the sine and cosine functions as a simple horizontal translation can also be seen.
• Recognising changes in amplitude, period, and phase	Using graphing technology, students explore the effects of the three control numbers on transforming the graph of $y=\sin x$ and $y=\cos x$. Examine the graphs of: • $y=A\cos x$ and $y=A\sin x$ • $y=\sin Bx$ and $y=\cos Bx$ • $y=\cos(x+C)$ and $y=\sin(x+C)$.
• Identifying contexts suitable for modelling by trigonometric functions and use them to solve practical problems	Students sketch the graphs of simple sinusoidal functions, to represent a range of different contexts.
• Solving trigonometric equations using technology and algebraically in simple cases	Students solve these equations both graphically (using technology) and algebraically for simple cases such as $\cos x=\frac{1}{2}$ or $\sin 2x=\frac{1}{2}$.
What special relationships can be observed by examining the sine and cosine functions and their behaviour in the unit circle? • $\sin(-x)=-\sin x$ • $\cos(-x)=\cos x$ • $\sin(x+\frac{\pi}{2})=\cos x$ • $\cos(x-\frac{\pi}{2})=\sin x$	Students consider the deduction of these useful identities by looking at the unit circle. They compare their graphs with those of other students to recognise, for example, change in amplitude. A more extensive exploration of trigonometric identities is required in Topic 10: Further trigonometry.

Key questions and key concepts

Where does the tangent function fit into all this?

- Understanding the relationship between the angle of inclination and the gradient of the line
- $\tan x = \dfrac{\sin x}{\cos x}$
- The graphs of the functions
  - $y = \tan x$
  - $y = \tan Bx$
  - $y = \tan(x + C)$

Considerations for developing teaching and learning strategies

The slope of the radius OP as P travels round the unit circle and generates the tangent function. Students investigate the behaviour of this function and its graph with a view to understanding ways in which it is different from, and similar to, the sinusoidal functions.

Students develop awareness of the construction that gives this function its name, as this offers another way of understanding the behaviour of the function for values between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$.

Determine the exact values of $\tan\theta$ for integer multiples of $\dfrac{\pi}{6}$ and $\dfrac{\pi}{4}$.

Topic 4: Counting and statistics

The study of inferential statistics begins in this unit with the introduction to counting techniques and the use of combinations for counting the number of selections from a group. An exploration of distributions and measures of spread, extending students' knowledge of the measures of central tendency in statistics, provides background for the study of inferential statistics in Stage 2 Mathematical Methods.

Subtopic 4.1: Counting

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can the number of ways something will occur be counted without listing all of the outcomes?	Calculations can sometimes involve working out the number of different ways in which something can happen. Since simply listing the ways can be tedious and unreliable, it is helpful to work out some techniques for doing this kind of counting.
How can the number of ways of making several different choices in succession be counted? • The multiplication principle • Factorials and factorial notation • Permutations	Students explore the multiplication principle by using tree diagrams and tables. For example, find the number of ways in which a three-course meal can be chosen from a menu. By looking at the number of arrangements, students can find the number of ways of arranging n different things is $n!$, leading to the number of permutations (or arrangements) of n objects taking r at a time $P_r^n = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$ Students solve problems involving the multiplication principle and permutations (using only discrete objects). For example, students determine the number of possible car number plates or the number of different ways in which five candidates in an election can be listed on a ballot paper.
How can the number selections be counted for different groups? • Understand the notion of a combination as an unordered set of distinct objects	Students work with ordered arrangements and unordered selections and use the multiplication principle to develop the link between the number of ordered arrangements and the number of unordered selections $r!\times$ number of unordered selections $= P_r^n$. Examples include counting the number of handshakes for a group of people, or the number of teams of three students that can be chosen from the five students who have nominated themselves.
The number of combinations (or selections) of r objects taken from a set of n distinct objects is C_r^n.	To find the number of combinations (or selections), first count the number of permutations (or arrangements) and then divide by the number of ways in which the objects can be arranged.

Key questions and key concepts	Considerations for developing teaching and learning strategies
Use $C_r^n = \frac{P_r^n}{r!} = \frac{n!}{(n-r)!r!}$ to solve problems.	For example, how many cricket teams of 11 players can be chosen from a squad of 14 players or, if an airline has 10 passengers on standby, how many ways could the airline choose the passengers to fill the remaining 4 seats?
Use the notation $\binom{n}{r}$ and the formula $$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$ for the number of combinations of r objects taken from a set of n distinct objects.	Alternatively, $$C_r^n = \binom{n}{r} = \frac{n(n-1)(n-2)...(n-r+1)}{r(r-1)(r-2)...1}.$$ Students use technology for calculating with larger numbers.
	Explore combinations related to the coefficients in the expansion of $(x+y)^n$ and Pascal's triangle: • Expand $(x+y)^n$ for integers $n = 1,2,3,4$ • Recognise the numbers $\binom{n}{r}$ as binomial coefficients (as coefficients in the expansion of $(x+y)^n$). It is useful to start with expanding: $(1+a) =$ $(1+a)(1+b) =$ $(1+a)(1+b)(1+c) =$ $(1+a)(1+b)(1+c)(1+d) =$ Organise the terms in order of the number of factors, then make $x = a = b = c = d$ to link with the notion of the coefficients in the expansion of $(1+x)^n$ as the number of combinations (unordered sets of objects).

Subtopic 4.2: Discrete and continuous random data

Key questions and key concepts	Considerations for developing teaching and learning strategies
How are discrete variables different from continuous variables? • Continuous variables may take any value (often within set limits); for example, height and mass • Discrete variables may take only specific values; for example, the number of eggs that can be purchased at a supermarket	Activities can illustrate how values that may be considered as fixed are actually variable, with random values; for example: • measuring the length of all the new pencils in a box • measuring the actual mass of different '1 kg' bags of potatoes. Students measure different variables (e.g. their height, mass, number of teeth) and classify them as discrete or continuous. Low resolution measuring devices (e.g. rulers that measure only down to centimetres) can be used to show that continuous variables may be recorded in a way that makes them appear to be discrete.

Subtopic 4.3: Samples and statistical measures

Key questions and key concepts	Considerations for developing teaching and learning strategies
	The emphasis in this subtopic is on reviewing the mean and developing an understanding of the standard deviation as a measure of spread.
What values are useful in describing the centre of a sample of data? • Briefly consider mean, median, and mode	Comparisons of different data sets can show the strengths and weaknesses of the different central tendencies.
What values are useful in describing the spread of a sample of data? • Consider range and interquartile range	Comparisons of different data sets can also show the strengths and weaknesses of the different measures of spread.
• Standard deviation of a sample gives a useful measure of spread, which has the same units as the data	Calculations of standard deviations of small samples using the formula $s=\sqrt{\frac{1}{N-1}\sum\left(x-\bar{x}\right)^2}$ illustrate what it calculates, and can be replaced with calculations using electronic technology once the concept is understood.

Subtopic 4.4 Normal distributions

Key questions and key concepts	Considerations for developing teaching and learning strategies
Why do normal distributions occur? • The value of the quantity is the combined effect of a number of random errors	Students are introduced to a variety of quantities whose variation is approximately normal (e.g. the volume of a can of soft drink or the lifetime of batteries). In investigating why normal distributions occur, students build a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers.
What are the features of normal distributions? • Bell-shaped • Position of the mean • Symmetry about the mean • Characteristic spread • Unique position of one standard deviation from the mean	A refined spreadsheet from this activity allows students to see the features of normal distributions develop (i.e. whatever the mean or standard deviation, all normal distributions have approximately 68% of the data one standard deviation on either side of the mean; approximately 95% is within 2 standard deviations; and approximately 99.7% is within 3 standard deviations).
Why are normal distributions so important? • Variation in many quantities occurs in an approximately normal manner, and can be modelled using a normal distribution How can the percentage of a population meeting a certain criterion be estimated for normal distributions?	Students calculate proportions or probabilities of occurrences within plus or minus integer multiples of standard deviations of the mean.

Topic 5: Growth and decay

This topic covers the study of exponential and logarithmic functions under the unifying idea of modelling growth and decay. Knowledge of indices enables students to consider exponential function and gain an appreciation of how exponential functions can model actual situations involving growth and decay.

The mathematical models investigated arise from actual growth and decay situations such as human population growth, the growth of bacteria, radioactive decay, and the spread of diseases. By developing and applying these mathematical models, students see how the wider community might use them for analysis, prediction, and planning.

So that actual data can be handled efficiently, technology is used extensively in this topic, for both graphing and calculation. Much of the technology has the facility to fit curves to data automatically. This allows students to compare their own models with a solution from another source.

Subtopic 5.1: Indices and index laws

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can indices be used to solve problems?	Briefly consider indices (including negative and fractional indices) and the index laws. Simplify algebraic products and quotients using index laws, applying knowledge of index laws to algebraic terms, and simplifying algebraic expressions using positive and negative integral indices, and fractional indices. Use radicals and convert to and from fractional indices. Surds occur when solving quadratic equations, using Pythagoras' theorem and trigonometry. Define rational and irrational numbers and perform operations with surds and fractional indices: • understanding that the real number system includes irrational numbers • extending the index laws to rational number indices • performing the four operations with surds.

Subtopic 5.2: Exponential functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is meant by an exponential relationship? Establish and use the algebraic properties of exponential functions.	Some examples are compound interest, depreciation, half-lives of radioactive material, simple population models (bacteria, locusts, etc.).
What kind of behaviours do exponential functions show? Recognise the qualitative features of the graph of $y = a^x (a > 0)$ and of its translations $y = a^x + b$ and $y = a^{x+c}$ and dilation $y = ka^x$.	Examine graphs of exponential functions involving powers of simple numbers such as 2, 10, and ½ (although others can be used). Technology can be used to explore different graphs. Emphasise the similarities that characterise these functions, such as asymptotes, intercepts, and behaviour as $x \to \pm\infty$.
How can problems that involve exponential functions be solved?	Determining the x-value for a given y-value (e.g. finding when a population should reach a certain value or finding the doubling time) can be done graphically, using technology to refine the answers, and algebraically in simple cases.

Subtopic 5.3: Logarithmic functions

Key questions and key concepts

How is it possible to get an exact solution to an equation where the power is the unknown quantity?

- Definition of the logarithm of a number
- Rules for operating with logarithms

  $\log_a b = x \Leftrightarrow a^x = b$ and the relationships

  - $\log_a a^x = a^{\log_a x} = x$
  - $\log_a mn = \log_a m + \log_a n$
  - $\log_a \frac{m}{n} = \log_a m - \log_a n$
  - $\log_a b^m = m \log_a b$
- Solving exponential equations, using logarithms (base 10)

Considerations for developing teaching and learning strategies

The study of logarithms arises from the need to be able to find an exact mathematical solution to exponential equations.

Discussion of the historical development of the technique, and its power in enabling mathematicians to solve a range of problems, is useful and interesting.

Problems involving determining the x-value for a given y-value in equations of the form $y = a^x$ (e.g. finding when a population should reach a certain value or finding the doubling time) are revisited and performed using logarithms (base 10).

New problems are posed in context to reinforce the necessary skills.

Topic 6: Introduction to differential calculus

The development of calculus enabled the study of the links between variables that are constantly changing. The use of mathematical modelling from other topics in Stage 1 Mathematics can be extended significantly by exploring rates of change using differentiation.

Rates and average rates of change are introduced, followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically, by calculating difference quotients both geometrically as slopes of chords and tangents, and algebraically. Calculus is developed to study the derivatives of polynomial functions and other linear combinations of power functions, with simple applications of the derivative to curve sketching, calculating slopes and equations of tangents, determining instantaneous velocities, and solving optimisation problems. The range of functions that can be differentiated and the different uses of derivatives are expanded in Stage 2 Mathematical Methods and Stage 2 Specialist Mathematics.

Subtopic 6.1: Rate of change

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is a rate of change? • A rate of change is a ratio of the change in one quantity compared with that in a second, related, quantity	This concept can be covered in the context of, for example, finding average speeds, costs per kilogram, litres of water used per day, or watts of power used per day. A constant rate of change can be identified through the exploration of examples such as running, driving at a steady rate, or leaving a mains electrical appliance operating for a period of time. This can be discovered: • numerically in a table with a constant adder • algebraically as a property of a linear function • graphically as the gradient of a straight line.
How can the rate of change of a non-linear function $f(x)$ over an interval be considered? • The average rate of change of function $f(a)$ in the interval from a to $a+h$ is $\dfrac{f(a+h)-f(a)}{h}$ • The average rate of change is interpreted as the slope of a chord	Using some of the contexts already mentioned, the concept of non-constant rate of change can be explored by considering average rates over different time intervals, for example: • an accelerating car • water delivered from a cask or dispenser (under gravity) • power delivered from a storage battery for a sufficient time to flatten the battery.

Subtopic 6.2: The concept of a derivative

Key questions and key concepts	Considerations for developing teaching and learning strategies
	This concept can be strengthened by working: • numerically from tables of data • algebraically from a formula • graphically (and geometrically) by considering gradients of chords across graphs of curves (graphics calculators, interactive geometry, and graphing software provide invaluable visual support, immediacy, and relevance for this concept). Applying all three approaches in one context strengthens the presentation of this concept. To aid progression to future subtopics, students explore how the average rate of change varies as the width of the interval decreases.
How can the rate of change at a point be approximated?	The rate of change across an interval is an approximation of the rate of change at a point (instantaneous rate of change). As the interval decreases, the approximation approaches the instantaneous rate of change (also to be interpreted as a chord approaching a tangent).
What is a limit?	The instantaneous rate of change of a function at a point is the limit of the average rate of change over an interval that is approaching zero. The notion of a limit can be developed by attempting to evaluate the fractions of the form $$\frac{(a+h)^2-a^2}{h}$$ as h approaches zero.
How can the instantaneous rate of change using derivatives be determined from first principles?	The derivative can be introduced as a summary of the concepts of rates of change and as a way to calculate an instantaneous rate of change.

Key questions and key concepts

Find the derivative function from first principles using $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

Introduce the alternative notation for the derivative of a function $f'(x)=\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Considerations for developing teaching and learning strategies

From first principles, find

- the derivative at a given point

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

- the derivative of a function

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

- the derivatives of functions such as $f(x)=x^2-6x$ at particular points and as functions
- the derivative of x^n for integer values of n as an introduction to the development of the rules of differentiation.

Subtopic 6.3: Computations of derivatives

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can the derivative of x^n where n is an integer, be found? Establish the formula $\frac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}$ when $y = x^n$ where n is an integer.	Estimate numerically the value of a derivative, for simple power functions.

Subtopic 6.4: Properties of derivatives

Key questions and key concepts	Considerations for developing teaching and learning strategies
Is the derivative a function?	Briefly consider the definition of a function.
What are the rules that apply to differentiation? Recognise and use the linearity of the derivative.	The use of differentiation by first principles for a number of examples of simple polynomials develops the rule $h'(x) = f'(x) \pm g'(x)$ for $h(x) = f(x) \pm g(x)$ which leads to $h'(x) = kf'(x)$ for $h(x) = kf(x)$.
Calculate derivatives of polynomials and other linear combinations of power functions.	

Subtopic 6.5: Applications of derivatives

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can differentiation be used to solve problems?	Students use functional models and their derivatives in the given contexts.
Solve problems that use polynomials and other linear combinations of power functions, involving the following concepts: • The slope and equation of a tangent • Displacement and velocity • Rates of change ◆ increasing and decreasing functions	Focus on position versus time graphs to describe motion where the velocity equates to the slope of the tangent at any point on the graph. Use a sign diagram to determine intervals in which the function is increasing or decreasing.
• Maxima and minima, local and global ◆ stationary points ◆ sign diagram of the first derivative ◆ end points	Use displacement functions and their first derivatives: object changes direction when velocity changes sign; object is at rest when velocity is zero.
• Optimisation	Examine optimisation problems involving simple polynomials and other linear combinations of power functions. Describe relationships between perimeter and area, area and volume, to minimise costs or optimise dimensions of three-dimensional objects. The following are examples of possible contexts: • Economics • Population dynamics • Energy consumption • Water use • Drug concentration.

Topic 7: Arithmetic and geometric sequences and series

Arithmetic and geometric sequences and series and their applications, such as growth and decay, are introduced and their recursive definitions applied.

Subtopic 7.1: Arithmetic sequences and series

Key questions and key concepts

Are there examples of sequences of numbers where there is a constant amount of increase (or decrease) in the values?

- Find the generative rule for a sequence, both recursive and explicit, using $t_{n+1} = t_n + d$ and $t_n = t_1 + (n-1)d$
- Determine the value of a term or the position of a term in a sequence
- Describe the nature of the growth observed
- $S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2t_1 + (n-1)d)$

Considerations for developing teaching and learning strategies

Students are presented with a variety of situations in which arithmetic sequences occur (e.g. simple interest).

The structure of a sequence (as a starting value continually augmented by a constant adder) can be clearly seen in the simple programs used to generate the terms on a calculator or computer.

Questions can be framed in the context of the growth situation being investigated.

Use graphs here, and emphasise links between the algebraic rule and the shape of the graph.

The sum of series is useful to find; for example, the number of seats in a section of a stadium.

Subtopic 7.2: Geometric sequences and series

Key questions and key concepts	Considerations for developing teaching and learning strategies
Are there examples of sequences of numbers where there is a constant ratio of increase (or decrease) in the values?	Some examples are compound interest, depreciation, half-lives of radioactive material, simple population models (bacteria, locusts, etc.).
• Recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$	
• Use the formula $t_n = r^{n-1}t_1$ for the general form of a geometric sequence and recognise its exponential nature	
• Understand the limiting behaviour as $n \to \infty$ of the terms t_n and its dependence on the value of the common ratio r	Use graphs and emphasise links between the algebraic rule and the shape of the graph.
• Establish and use the formula $S_n = t_1 \dfrac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence	Finding the total value of the investment after a given number of periods requires summing this sequence. At this point the general formula for the sum of a geometric sequence can be derived and used to answer questions such as 'How much will I have after ...?' or 'How much do I need to put away each month?' or 'How long will it take me to save ...?'.
• If $\lvert r \rvert < 1$ then $S_n \to \dfrac{t_1}{1-r}$ as $n \to \infty$	Investigate the consequence of $\lvert r \rvert < 1$: $$1 + r + r^2 + ... + r^n = \frac{1 - r^{n+1}}{1 - r},$$ and if $\lvert r \rvert < 1$ then the right-hand side tends to $\dfrac{1}{1-r}$ as $n \to \infty$.

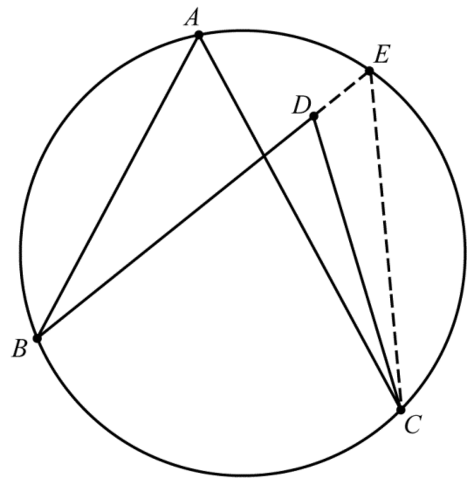
Topic 8: Geometry

The context of this topic is the geometry of planar figures. The focus is on forming and testing hypotheses about their properties, which if proved to be true become theorems. Students form ideas about which properties of a figure might be universal, and test enough examples to be convinced that their idea is correct before they attempt a formal proof. For testing to be effective and efficient, electronic technology is used whenever possible.

Subtopic 8.1: Circle properties

Key questions and key concepts	Considerations for developing teaching and learning strategies
What properties are found when angles, lines, and polygons are drawn on and within circles? • Chord and tangent properties ◆ Radius and tangent property ◆ Angle between tangent and chord (alternate segment theorem) ◆ Length of the two tangents from an external point • Properties of angles within circles ◆ Angle subtended at the centre is twice the angle subtended at the circumference by the same arc ◆ Angles at the circumference subtended by the same arc are equal ◆ Opposite angles in a cyclic quadrilateral are supplementary ◆ An angle in a semicircle is a right angle ◆ Chords of equal length subtend equal angles at the centre ◆ Converses of the above properties ◆ Intersecting chords theorem, including internal and external intersections, and the special case of a tangent and chord through an external point	Questions can be posed to guide the investigation of the properties of a circle; for example: • How can a right angle be marked out with no measuring equipment other than some pegs and lengths of string? • In a penalty shot or conversion attempt in rugby union, where is the best place to stand on a circular arc to get the widest angle of attack at the goal? Or, where is the best place to sit in a row at the theatre? • Can a circle always be inscribed round a rectangle? How? Can this be done with any other kinds of parallelograms? Why not? What kinds of other quadrilaterals do have an inscribing circle? Do they have special properties?

Subtopic 8.2: The nature of proof

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can the properties discovered by investigation be proved?	The hypotheses developed when answering questions about circle properties could be tested first by trial and error, preferably using electronic technology.
• Justification of properties of circles	Once this testing process has been completed for each property, students are led through a set of logical steps that justify the property for all cases. The nature of proof: • Note that use of similarity and congruence is required in some proofs • Use implication, converse, equivalence, negation, contrapositive • Use examples and counter-examples • Use proof by contradiction, for example, to prove that 'Angles at the circumference subtended by the same arc are equal'. That is, consider three concyclic points A, B, and C with a fourth, E, on the circle as shown:  Given $\angle BAC = \angle BDC$, prove D must be at position E.

Topic 9: Vectors in the plane

The study of vectors in the plane provides new perspectives for working with two-dimensional space. Vectors are used to specify quantities that have size (magnitude) and direction. These quantities include velocity, force, acceleration, displacement, and are used in fields such as physics and engineering. The topic includes vector operations, their applications, and their use in proving results in geometry.

Subtopic 9.1: Vector operations

Key questions and key concepts	Considerations for developing teaching and learning strategies
Representing vectors in the plane by directed line segments What are the rules for vector operations? • Vector addition and subtraction • Scalar multiples of a vector • Applications of scalar multiples: parallel vectors, ratio of division	Define magnitude and direction of a vector. Examples include displacement and velocity. Drawing software provides an excellent environment for manipulating vectors and appreciating the triangular nature of vector addition. The problem of moving a robot from A to B, using only the repetition of a small number of possible movements (in vector form), reinforces the concept that the order of addition changes the path followed but not the resultant vector. In the same context, the concept of scalar multiples as several steps using the same vector is made clear. Similarly, negative multiples simply involve moving backwards along an arrow.

Subtopic 9.2: Component and unit vector forms

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can a vector be described in the Cartesian plane?	In Subtopic 9.1, students have operated with vectors without reference to a grid system that requires the vectors to be defined in component form.
• Use ordered-pair notation and column vector notation • Convert a vector into component and unit vector forms	Students practise vector addition and scalar multiplication. They explore the concept of component vectors, column vectors, combinations of unit vectors, and position vectors.
• Determine length and direction of a vector from its components	Concepts from physics can be explored, for example, the effects on horizontal motion of forces from different directions and the analysis of static systems (in conjunction with force table experiments if the equipment is available).

Subtopic 9.3: Projections

Key questions and key concepts	Considerations for developing teaching and learning strategies
How much of one vector is operating in the direction of another?	In Subtopic 9.2, students concentrated on the x and y components of a vector, which are projections of the vector onto the axes. In this subtopic, students work out the projection of one vector onto another. For instance, how much help does an aeroplane travelling north gain from a south-westerly wind, and what are the consequences for fuel consumption? How fast can a person sail on a particular course in a specified wind and with the tide running? Shadows also provide a practical context for this work.
• The dot (scalar) product • The angle between two vectors • Perpendicular vectors • Parallel vectors	Calculating the projection algebraically leads to the definition of the dot product and the formula for the cosine of the angle between the two vectors. From here, the implications of a zero dot product and other properties can be investigated.

Subtopic 9.4: Geometric proofs using vectors

Key questions and key concepts	Considerations for developing teaching and learning strategies
Can vector concepts be used to prove results in geometry? Geometric proofs using vectors in the plane include: • The diagonals of a parallelogram meet at right angles if and only if it is a rhombus • Midpoints of the sides of a quadrilateral join to form a parallelogram • The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides	Understanding of scalar multiples and the dot product provides a powerful tool for proving results about parallelism and perpendicularity.

Topic 10: Further trigonometry

In this topic, students extend their understanding of trigonometric functions. Students model circular motion in the familiar contexts of, for example, Ferris wheels, merry-go-rounds, and bicycle wheels. These functions are fundamental to understanding many natural oscillatory phenomena such as lunar illumination, tidal variation, and wave propagation.

Subtopic 10.1: Further trigonometric functions

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can periodic phenomena be modelled mathematically?	Note that Topic 3, Subtopic 3.3 also considers trigonometric functions and must precede this topic.
• The general function $y = A\sin B\left(x - C\right) + D$ • Extend to: $y = A\cos B\left(x - C\right) + D$ $y = A\tan B\left(x - C\right) + D$	Using graphing technology, students can explore the effects of the four control numbers (individually and in combination) in the general sinusoidal model $y = A\sin B(x - C) + D$ on transforming the graph of $y = \sin x.$ Students explore fitting functions of this form to their data from 'circle' examples such as the Ferris wheel. They relate the values they find for A, B, C, and D back to the parameters given in the real situation.
• Sketch graphs of sinusoidal functions	Having explored the effects of the four parameters, students sketch the graphs of sinusoidal functions without using technology (except to check their answers).
• Solve trigonometric equations of the form $y = k$ (where y is one of the functions above), finding all solutions	Students use the resulting equations to answer questions such as 'How high off the ground will you be if the wheel stops after 3 minutes?' or 'For how long in each revolution is a person able to see over the 3-metre fence round the Ferris wheel?' Students solve these equations both graphically (using technology) and algebraically.

Subtopic 10.2: Trigonometric identities

Key questions and key concepts	Considerations for developing teaching and learning strategies
What special relationships can be observed by examining the sine and cosine functions and their behaviour in the unit circle? • $\sin(-x), \cos(-x)$ $\sin^2 x + \cos^2 x,\ \sin 2x$ $\cos 2x,\ \sin\frac{1}{2}x,\ \cos\frac{1}{2}x$ in terms of $\sin x,\ \cos x$ • $\cos(A \pm B)$, and hence $\sin(A \pm B)$ in terms of $\sin A,\ \cos A,\ \sin B,\ \cos B$	Students are guided through the deduction of many of these useful identities by looking at the unit circle. They discover others by comparing their graphs. The formula for $\cos(A-B)$ can be derived from the unit circle, using the cosine rule. The other angle sum formulae follow from it, using the identities already learnt. Students prove these basic identities and apply them algebraically, to establish such results as $\cos 3x = 4\cos^3 x - 3\cos x.$
• Conversion of $A\sin x + B\cos x$ into the form $k\sin(x+\alpha)$	Students derive the results: $k > 0, k = \sqrt{A^2 + B^2},\ \cos\alpha = \frac{A}{k},\ \sin\alpha = \frac{B}{k}$
• The reciprocal trigonometric functions: $\sec\theta,\ \mathrm{cosec}\theta,\ \cot\theta$	Working from their definitions, students sketch graphs and simple transformations of these new reciprocal functions.

Topic 11: Matrices

Matrices provide new perspectives for working with two-dimensional space. The study of matrices includes extension of matrix arithmetic to applications such as linear transformations of the plane, solving systems of linear equations and cryptography.

Subtopic 11.1: Matrix arithmetic

Key questions and key concepts	Considerations for developing teaching and learning strategies
What is a matrix? • Order of matrices	Matrices can provide a useful representation of information in a wide range of contexts. From a given context students put numerical information in tabular form with the rows and/or columns labelled to identify what the elements represent.
What operations can be applied to matrices? • Addition and subtraction • Scalar multiplication • Matrix multiplication	Matrix addition is both commutative and associative. Matrix multiplication is associative but not commutative. Matrices can be multiplied only under certain conditions.
The identity matrix for matrix multiplication	The identity matrix I should be introduced via its properties: $AI = IA = A.$
What is the inverse of a square matrix?	The matrix A^{-1} is defined by the property $A^{-1}A = AA^{-1} = I$ Not all square matrices have an inverse; those which do not are called singular. The inverse is not defined for non-square matrices.
• Calculating the inverse of a 2×2 matrix when it exists	The formula $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ for a matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
• The determinant of a 2×2 matrix and its significance	The number $ad-bc$ is called the determinant of A and is denoted $\det A$. If $\det A = 0,$ the inverse does not exist.
How can matrix inverses be used? • Find the unique solution to matrix equations of the form $AX = B$ or $XA = B$ if it exists	For the system $AX = B,$ if A^{-1} does exist, then $X = A^{-1}B.$

Subtopic 11.2: Transformations in the plane

Key questions and key concepts	Considerations for developing teaching and learning strategies
What are some applications of matrices?	
Transformations in the plane and their description in terms of matrices. • Translations and their representation as column vectors, that is, 2×1 matrices • Define and use basic linear transformations • Consider dilations of the form $(x,y)\to(\lambda_1 x,\lambda_2 y)$, rotations about the origin where $\begin{bmatrix}x\\y\end{bmatrix}\to\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$, and reflection in a line which passes through the origin where $\begin{bmatrix}x\\y\end{bmatrix}\to\begin{bmatrix}\cos(2\theta) & \sin(2\theta)\\ \sin(2\theta) & -\cos(2\theta)\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$ • Apply transformations to points in the plane and geometric objects • Define and use composition of linear transformations and the corresponding matrix products • Establish geometric results by matrix multiplications • Show that the combined effect of two reflections in lines through the origin is a rotation	The representation of points in the plane as a coordinate pair can be considered as an example of matrix notation (1×2 for a row vector and 2×1 for a column vector).
Can transformations be 'undone'?	
• Define and use inverses of linear transformations and the relationship with the matrix inverse • Examine the relationship between the determinant and the effect of a linear transformation on area	An example is that of matrix codes: arithmetic via a matrix and decrypting via its inverse, using modulo 26. Not every 2×2 matrix will be suitable. The value of the determinant needs to be considered to ensure that decryption is possible.
• Note that if the determinant of a matrix is zero, then the corresponding transformation has no inverse	An example with no inverse: Projection on an axis e.g. $\begin{bmatrix}x\\y\end{bmatrix}\to\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$

Topic 12: Real and complex numbers

This topic is a continuation of students' study of numbers. Mathematical induction is introduced as a way of proving a given statement for all integers. Complex numbers extend the concept of the number line to the two-dimensional complex plane. This topic introduces operations with complex numbers, their geometric representation, and their use in solving problems that cannot be solved with real numbers alone.

Subtopic 12.1: The number line

Key questions and key concepts	Considerations for developing teaching and learning strategies
The number line represents all real numbers. What are some properties of special subsets of the reals? • Rational and irrational numbers	Consider surds and their operations: • Express rational numbers as terminating or eventually recurring decimals and vice versa. • Prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$.
• Proving simple results involving numbers	Some examples include: • The sum of two odd numbers is even. • The product of two odd numbers is odd. • The sum of two rational numbers is rational.
Is there a convenient notation for 'pieces' of the number line? • Interval notation	Use of square brackets and parentheses to denote intervals of the number line that include or exclude the endpoints. For example, the set of numbers x such that $a < x \leq b$ is denoted $(a, b]$.

Subtopic 12.2: Introduction to mathematical induction

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can a statement concerning all positive integers be proved?	This topic can be introduced through well-known analogies such as climbing a ladder or knocking down dominoes. Examples may be found online, such as D. J. Litman's http://people.cs.pitt.edu/~litman/courses/cs441/lecture15.pdf 'Dominoes and Mathematical Induction', https://www.youtube.com/watch?v=ZpPk99D_Vlg
An introduction to proof by mathematical induction: • understand the nature of inductive proof including the 'initial statement' and inductive step	Formal proofs of simple examples are expected, i.e. Let there be associated with each positive integer n, a proposition $P(n)$. If $P(1)$ is true, and for all k, $P(k)$ is true implies $P(k+1)$ is true, then $P(n)$ is true for all positive integers n. • prove results for simple sums, such as $1+4+9+...+n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n • prove results for arithmetic and geometric series.

Subtopic 12.3: Complex numbers

Key questions and key concepts	Considerations for developing teaching and learning strategies
Why were complex numbers ‘invented’?	
Define the imaginary number i as a solution to the quadratic equation $x^2+1=0.$	Analogy can be drawn with the extension of the natural numbers to the integers, integers to rationals, rationals to reals — in each case with a view to ensuring that certain kinds of equations have solutions. A parallel is drawn with the arithmetic of surds and how surds arise — for example, solving the quadratic equation $x^2+2x-1=0$ leads to numbers of the form $\left\{a+b\sqrt{2}: a, b \text{ rational}\right\}$.
Operations with complex numbers: • Real and imaginary parts • Complex conjugates • Arithmetic with complex numbers • $\mathrm{i}^2=-1$	• Define the imaginary number as a root of the equation $x^2=-1.$ • Use complex numbers in the form $a+b\mathrm{i}$ where a and b are the real and imaginary parts. • Determine and use complex conjugates; for $z=a+b\mathrm{i},\ \bar{z}=a-b\mathrm{i}.$ • Perform complex-number arithmetic: addition, subtraction, multiplication, and division. Students can add and subtract complex numbers using the usual rules of arithmetic and algebra. Multiplication calls for the same approach, with the additional need to simplify i^2 using the fact that $\mathrm{i}^2=-1.$ Division of complex numbers can be introduced by presenting a complex product such as $(2-\mathrm{i})(1+\mathrm{i})=3+\mathrm{i},$ inferring the result for $\dfrac{3+\mathrm{i}}{2-\mathrm{i}}$ and then asking: How can this result be obtained through calculation?

Subtopic 12.4: The complex (Argand) plane

Key questions and key concepts	Considerations for developing teaching and learning strategies
How can complex numbers be represented geometrically? • Cartesian form on the Argand diagram	The Cartesian plane as extension of the real number line to two dimensions. Correspondence between the complex number $a+b\mathrm{i}$, the coordinates (a, b) and the vector $[a, b]$.
• Vector addition in the complex plane	Complex-number addition corresponds to vector addition via the parallelogram rule. Complex numbers can be regarded as two-dimensional vectors with rules for multiplication and division to supplement addition and subtraction.
• Locating complex conjugates in the complex plane (Argand diagram)	Relative positions of $z=a+b\mathrm{i}$ and its conjugate; their sum is real and difference purely imaginary.
• Modulus	Recognising that $\lvert z\rvert=\sqrt{a^2+b^2}$ represents the length of a complex number when represented as a vector.

Subtopic 12.5: Roots of equations

Key questions and key concepts	Considerations for developing teaching and learning strategies
What has been the advantage of introducing complex numbers?	
• The introduction of i enables the solution of all real quadratic equations and the factorisation of all quadratic polynomials into linear factors	Consider the formula for solution of quadratic equations, with emphasis on arithmetic involving i. When the solutions of a real quadratic equation are complex, they are conjugates.

ASSESSMENT SCOPE AND REQUIREMENTS

Assessment at Stage 1 is school based.

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 1 Mathematics:

- Assessment Type 1: Skills and Applications Tasks
- Assessment Type 2: Mathematical Investigation.

For a 10-credit subject, students should provide evidence of their learning through four assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least two skills and applications tasks
- at least one mathematical investigation.

For a 20-credit subject, students should provide evidence of their learning through eight assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least four skills and applications tasks
- at least two mathematical investigations.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by teachers to:

- clarify for students what they need to learn
- design opportunities for students to provide evidence of their learning at the highest level of achievement.

The assessment design criteria consist of specific features that:

- students need to demonstrate in their evidence of learning
- teachers look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, gives students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships.

CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.

CT3 Application of mathematical models.

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation of mathematical results.

RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.

RC3 Use of appropriate mathematical notation, representations, and terminology.

RC4 Communication of mathematical ideas and reasoning to develop logical arguments.

RC5 Development and testing of valid conjectures.*

* In this subject students must be given the opportunity to develop and test conjectures in at least one assessment type.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

For a 10-credit subject, students complete at least two skills and applications tasks.

For a 20-credit subject, students complete at least four skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:

- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representations throughout the task. Electronic technology may aid and enhance the solution of problems. The use of electronic technology and notes in the skills and applications task assessments is at the discretion of the teacher.

Skills and applications tasks may provide opportunities to formulate and test conjectures.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation

For a 10-credit subject, students complete at least one mathematical investigation.

For a 20-credit subject, students complete at least two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop contexts, themes, or aspects of their own choice. Teachers should give some direction about the appropriateness of each student's choice, and guide and support students' progress in a mathematical investigation.

A mathematical investigation may provide an opportunity for students to work collaboratively to achieve the learning requirements. If an investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Teachers may need to provide support and clear directions for the first mathematical investigation. Where students undertake more than one investigation, subsequent investigations could less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. Computer Algebra Systems, spreadsheets, statistical packages) to assist in their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, technological skills, and results are important considerations.

Students complete a report on the mathematical investigation. In the report, they interpret and justify results, draw conclusions, and give appropriate explanations and arguments. The mathematical investigation may provide an opportunity to develop and test conjectures.

In the report, they formulate and test conjectures, interpret and justify results, draw conclusions, and give appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including:
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem.

A bibliography and appendices, as appropriate, may be used.

The format of an investigation report may be written or multimodal.

Each investigation report, excluding bibliography and appendices if used, must be a maximum of eight A4 pages if written, or the equivalent in multimodal form. The maximum page limit is for single-sided A4 pages with minimum font size 10. Page reduction, such as two A4 pages reduced to fit on one A4 page, is not acceptable. Conclusions, interpretations and/or arguments that are required for the assessment must be presented in the report, and not in an appendix. Appendices are used only to support the report, and do not form part of the assessment decision.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers refer to in deciding how well students have demonstrated their learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of a subject, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- taking into account the weighting given to each assessment type
- assigning a subject grade between A and E.

Performance Standards for Stage 1 Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Some development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly effective development and testing of valid conjectures.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures.

	Concepts and Techniques	Reasoning and Communication
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, and their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement in the school assessment are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 1 are available on the SACE website (www.sace.sa.edu.au).

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).